Identification of Hardspots in Mathematics At +2 level and Suggestions on Learning Strategies for Teachers/Junior College Lecturers of Kerala and Andhra Pradesh

Report

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Preface

The mathematics textbooks at +2 level in the Southern States have been revised by introducing some new concepts. Teachers expressed their difficulty in dealing with these new concepts. Accordingly, the state functionaries of A P and Kerala have requested Regional Institute of Education, Mysore to develop suitable materials dealing with the concepts so as to help the teachers. In this context, the coordinators have identified the hardspots through a questionnaire and with the personal contact of the teachers in Andhra Pradesh and Kerala. Since the contents of the textbooks of Kerala is a subset of the contents of the textbooks of Andhra Pradesh, this material is mainly prepared based on the mathematics textbooks of Andhra Pradesh. Based on the identified common hardspots, this material is being developed in a five-day workshop at Regional Institute of Education, Mysore.

In conclusion, the coordinator wants to express his deep gratitude to the faculty of NCERT, RIEM, SCERT and to the Lecturers and Principals of Govt and Private Junior Colleges in Andhra Pradesh and Kerala and to the staff of the Computer Processing Unit of RIEM for their direct and indirect help in bringing out this material.

(B S P Raju)
Academic Coordinator

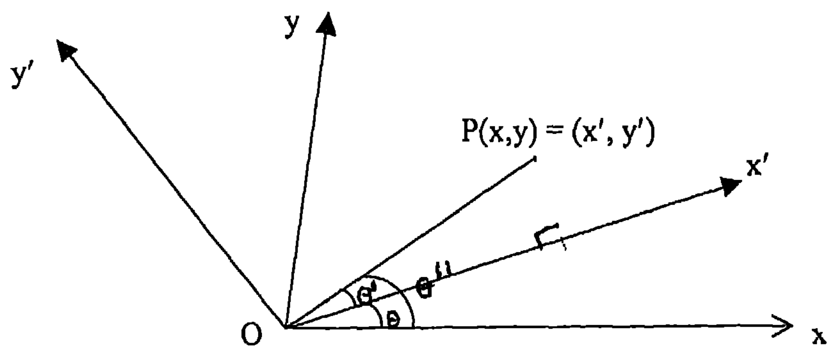
INTERMEDIATE FIRST YEAR

COORDINATE GEOMETRY

Reference Unit	Page No	Recommendations
0.1.2	Page 2	The signs of co-ordinates may be explained as I quadrant XOY : x is +ve y is +ve II quadrant X^1OY : x is –ve y is +ve III quadrant X^1OY^1 : x is -ve y is -ve IV quadrant XOY^1 : x is +ve y is -ve
1.1.1	Page 14 Definition	To understand the concept the definition can be modified as: Locus of a point is the path traced by the point which moves under certain geometric conditions.
1.2	Page 15	Equation of a locus may be explained as: The geometrical condition satisfied by a point (x,y) on a locus can be expressed algebraically as a relation between x and y which is called the equation of the law.
2.2.2	Page 23	Instead of getting (A) and obtaining (B) we can arrive at (B) straight away.
3.1	Page 35 Theorem	Avoid using sec α and cosec α
3 2.1	Page 36 Theorem	Deduce as a special case of point-slope form.
3.2.2	Page 37 Note	May be deleted Parametric form of a straight line may be given in the form $x=x_1+(x_2-x_1)\,t$ $y=y_1+(y_2-y_1)\,t$ [Generalized result in space is in the same format]
3.3.1	Page 41 Theorem	May be obtained from 4.1.4 Note Page 57
4 3.6	Page 66 Theorem	Statement can be given in the form $L_1+\lambda L_2=0$ $\lambda \in R$.
4.4.5	Page 71 Theorem	In equation use the parameter 't' instead of 'r'

Reference Unit	Page No	Recommendations
5.1.1	Page 86	The proof can be simplified by starting with two linear equations.
6.1.1	Page 99 6.1.1 Theorem Second Method	(3) and (4) can be replaced by $y=m_1x + c_1$ $y=m_2x + c_2$ to simplify the proof
6.1.4	Page 100 Theorem	(2) and (3) can be replaced by $y=m_1x +c_1$ $y=m_2x +c_2$
6.1.4	Page 102 Theorem	(2) and (3) can be replaced by $y=m_1x +c_1$ $y=m_2x +c_2$
7.2.6	Page 119 Theorem	Statement can be modified as: "The coordinates of the point which divides the line segment joining the two points $(x_1y_1z_1)$, $(x_2y_2z_2)$ in the ratio $\alpha:\beta$ $(\alpha+\beta=1)$ etc
7.2.16	Page 123 Theorem	Proof can be simplified by taking α *and* $\beta(\alpha+\beta=1)$ instead of λ and μ.
8.3.12	Page 144 Example	Too many parameters may be deleted.
9.1.2	Page 158 Theorem	Proof can be modified.
9.2 1	Page 161 Theorem	Instead of dealing with A,B,C,D we can consider three equations and eliminate A,B,C
9.3.5	Page 165 Theorem	To simplify club up 9.3 10 with 9.3.5
9.3.11 9.3.12	Page 166 Note Note	By putting $Z=$ const in the linear equation we get the result.
9.3.13	Page 167 Theorem	Proof can be modified.

Reference Unit	Page No	Recommendations
9.3.16	Page 168 Theorem	From (1) get the result from 9.1.7
9.3.21	Page 169 Example	Since the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ is parallel to x-axis The direction ratios satisfy the equation $\frac{1}{a}(1)+\left(\frac{1}{b}\right)(0)+\left(\frac{1}{c}\right)(0)=0$ $\therefore \frac{1}{a}=0$ Substituting we get the result.
2.2 2	Page 23	Exemplar material providing alternate treatment to the ones given in the textbook.



EXPLANATION

$X'\hat{O}X=\theta \quad P\hat{O}X'=\theta' \quad P\hat{O}X=\theta'' \quad \theta''=\theta'+\theta$

$X' = OM$

$= OP \text{ Cos } \theta'$

$= OP \cos(\theta'' - \theta)$

$= OP \cos\theta'' \cos\theta + OP \sin\theta'' \sin\theta$

$= OL \cos\theta + PL \sin\theta$

$= x\cos\theta + y\sin\theta$

Similarly for y'.

Reference Unit	Page No	Recommendations
3.1	Page 35 Theorem	Write OA = $\frac{p}{\cos\alpha}$ $OB = \frac{p}{\sin\alpha}$ The equation reduces to $\frac{x}{\frac{p}{\cos\alpha}} + \frac{y}{\frac{p}{\sin\alpha}} = 1$ $X\cos\alpha + y\sin\alpha = p$
5.1.1	Page 83 Theorem	Let $b \neq o$ Consider the equations $y-m_1x=0$, $y-m_2x=0$ $(y-m_1x)(y-m_2x)=0$ $\Rightarrow m_1m_2x^2 - (m_1+m_2)xy + y^2=0$ - (1) This represents a pair of straight lines passing through the origin and having slopes m_1 and m_2. Taking $m_1m_2 = \frac{a}{b}$ $m_1+m_2 = \frac{-2h}{b}$ Equation (1) reduces to $\frac{a}{b}x^2 + \frac{2h}{b}xy + y^2 = 0$ $\therefore ax^2 + 2hxy + by^2=0$ (2) Equation (2) represents a pair of lines passing through the origin. In case b=0, the equation reduces to $x(ax + 2hy) = 0$ i.e, $x=0$ $ax+2hy=0$.
6.1.1	Page 99 Theroem	Consider $(y-m_1x-c_1)(y-m_2x-c_2)$ $= m_1m_2x^2 - (m_1+m_2)xy + y^2 + (m_1c_2+m_2c_1)x - cc_1+c_2)$ $y + c_1c_2=0$ so that $a=m_1m_2$ $b=1$ $c=c_1c_2$ $2g= m_1c_2 + m_2c_1$ $2f = -(c_1+c_2)$ $2h = -(m_1+m_2)$ $(2f)(2g)(2h) = (c_1+c_2)(m_1c_2+m_2c_1)(m_1+m_2)$ $= m_1m_2(c_1=c_2)^2 + [(m_1c_2+m_2c_1)^2 - 2m_1m_2c_1c_2)] + c_1c_2[(m_1+m_2)^2 - 2m_1m_2]$ $= a(2f)^2 + b((2g)^2-2ac) + c((-2h)^2-2ab)$ $\because 8fgh = 4af^2 + 4bg^2 + 4ch^2 - 4abx$ which reduces to $abc + 2fgh - af^2 - bg^2 - ch^2=0$

Reference Unit	Page No	Recommendations
7.2.16	Page 123 Theorem	Let R (x,y,z) be any point on the line $X=\overleftrightarrow{PQ}$ $X=\alpha x_2+\beta x_1$ $y=\alpha y_2+\beta y_1$ $z=\alpha z_2+\beta z_1$, $\alpha+\beta=1$ $\alpha=t$, $\beta=1-t$ $x= x_1+ (x_2-x_1)\, t.$ $y= y_1 + (y_2-y_1)\, t$ $z= z_1 + (z_2-z_1)\, t.$
9.1 2	Page 158 Theorem	Let R be a point on the line $\overleftrightarrow{PQ}$ then there are numbers m and n with m+n=1 and $\alpha=mx_1+nx_2$ $\beta=my_1+ny_2$ $\gamma=mz_1+nz_2$ consider $A\alpha + B\beta + C\gamma + D$ $=A(mx_1 + nx_2) + B(my_1 + ny_2) + C(mz_1 + nz_2) + D$ $= m (Ax_1+By_1+Cz_1+D) + n (Ax_2+By_2+Cz_2+D)$ $=0.$
9.1.3	Note	Illustrations can be given by considering simple cases like xy-plane (z=0), planes parallel to xy-plane (z=k), yz plante etc. State the converse as: any given plane can be represented by equation (1).
9.2.1	Page 161 Theorem	(1) - (2) . $A(x-x_1) + B(y-y_1) + C(z-z_1)=0$ (3) - (2) : $A(x_2-x_1) + B(y_2-y_1) + C(z_2-z_1)=0$ (4) - (2) : $A(x_3-x_1) + B(y_3-y_1) + C(z_3-z_1)=0$ Eliminating A,B,C we get $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$

Reference Unit	Page No	Recommendations
9.3.13	Page 167 Theorem	If a,b,c are direction ratios of the normal to the plane then the direction cosines are $\frac{a}{\sqrt{a^2+b^2+c^2}}$, $\frac{b}{\sqrt{a^2+b^2+c^2}}$, $\frac{c}{\sqrt{a^2+b^2+c^2}}$ $\frac{ax}{\sqrt{a^2+b^2+c^2}}+\frac{by}{\sqrt{a^2+b^2+c^2}}+\frac{cz}{\sqrt{a^2+b^2+c^2}}=p$ where $p = -\frac{d}{\sqrt{a^2+b^2+c^2}}$ $d>0$ $= \frac{d}{\sqrt{a^2+b^2+c^2}}$ $d<0$ $\therefore$ ax+by+cz+d=0
9.3.14	Theorem	The equation of the plane parallel to ax+by+cz+d=0 is ax+by+cz+d^1=0 Since this plane passes thro' (x_1,y_1,z_1) we have, $a(x-x_1) + b(y-y_1) + c(z-z_1)=0$.

INTERMEDIATE FIRST YEAR

CALCULUS

Chapter 1 : Functions, Limits and Continuity

Reference Unit	Page No	Article	Recommendations	
1.01 1.1	1 to 7	Functions Real Functions – Graphs	1.	The entire unit may be deleted because the unit has been covered in greater detail in the First Year Maths – Algebra and Trigonometry. Chapter 1 : Functions and Mappings.
1.2		Limits	1.	The chapter may be preceded by recalling concepts as function, Domain/ range and graphs of functions. Intervals of definition and some standard examples.
1.2.1	18		2.	\the definition of 'limit' may be made clear. Additional explanation of technical terms and symbols (like $\in$ - δ terminology).
1.2 3	19		3.	Worked example to be solved avoiding the $\in$ - δ approach.
			4.	Lengthy/ difficult examples on 1.2.4 to be deleted.
Ex.1 (b)	23		5.	Delete exercises 1 to 6
1.3	24-25		6.	In the statement of theorems, the language is to be made non-technical (like removing phrases 'deleted n-hood of a').
	26-28		7.	Proofs of theorems 1.3.11, 1.3.12, 1.3.13, 1.3.14 may be deleted. Only statements of theorems be retained.
	27		8.	An easy proof for theorem 1.3.17. (Using Binomial theorem) may be given (deleting the proof found in the page).

Reference Unit	Page No	Article		Recommendations
1.4 : 1.4.1 & 1.4.2	32-33		9.	Delete the $\in$ - δ language and define the left/right limits.
1.4.3	33		10.	Restate the theorem as – If $\lim\limits_{x \to a-} f(x) = \lim\limits_{x \to a^+} f(x) = l$ (*say*) *then* $\lim\limits_{x \to a} = l$
1.5	35	Infinity, Limits at infinity	11.	Avoid the δ - language in the definitions. Instead take examples to illustrate the concepts.
1.6 : 1.6.1	38	Continuity		Definition : $\lim\limits_{x \to a} f(x) = f(a) \Leftrightarrow$ continuity of f(x) at x = a. Geometrical meaning of continuity/ discontinuity to be given for clarity and better understanding.
	39			1.6.3 may be deleted.

Overall recommendations on the Chapter 1 : [P.No. 1 to 41]

1. The unit on functions and graphs (1.01 and 1.1) may be deleted as it has been discussed in the other book in detail).
2. The language to be modified by avoiding the technical language as $\in$ - δ phrases.
3. As supportive approach for better clarity, graph/diagrammatic treatment is necessary.
4. Proofs of certain theorems to be (a) deleted, (b) made easier.

Chapter 2 : Differentiation

Reference Unit	Page No	Article	Recommendations	
2.1	42	Differentiation	1.	In addition to the analytical definition of $\frac{dy}{dx}$, (a) the geometrical meaning, and (b) an instantaneous rate of change of y w.r.t. x may also be given, for better understanding.
2.2.8	51			The proof of the chain rule may be *made easier.*
	50			The proof of theorem 2.2.7 (the derivative of the reciprocal of a function) ***is to be deleted.***
	51			The proof of the theorem 2.2.10 *to be deleted.*

Overall Recommendations on Chapter 2 [P.42 to 82] :

1. Proofs of certain theorems (as indicated) may be deleted.
2. Certain proofs can be replaced by non-rigorous proof for easy comprehension.
3. Even problems which are solved may be solved by methods which are simpler, alternative ways, containing fewer steps (avoiding lengthy answers).
4. The table of Standard Derivative may be given as a useful reference.

Chapter 3 : Successive Differentiation

Refe-rence Unit	Page No	Article	Recommendations	
3.1.18	87		1.	Such problems in which De Moivre's Thoerem/factorization into complex factors are used, may be deleted.
3.1.21	89		2.	The entire unit on L-Thm and problems thereon may be deleted, as it is going to be an unnecessary burden on the students.

Overall Recommendations on Chapter 3 [P.82 to 99] :

1. Proofs of certain theorems (as indicated) may be deleted.
2. Certain proofs can be replaced by non-rigorous proof for easy comprehension.
3. Even problems which are solved may be solved by methods which are simpler, alternative ways, containing fewer steps (avoiding lengthy answers).
4. The table of Standard Derivative may be given as a useful reference.

Chapter 4 : Differentiation – Applications

Reference Unit	Page No	Article	Recommendations	
4.3	108	Definitions	1.	The definitions of increasing/decreasing functions (a) at a point, (b) over an interval may be made.
4.3.1 & 4.3.2	108		2.	Simpler and intelligible by avoiding $\in$ - δ language and supplementing by geometrical approach. In one of the increasing function, the graph is a rising one and .... decreasing .... Falling one.
4.3.3	108			Proof may be replaced by simpler ones – Geometrical approach.
4.4	113			**Maxima/Minima** : Geometrical approach is recommended in addition to the treatment given. Case of $f'(x) = 0$, $f''(x) = 0$ needs to be discussed. A word about a point of inflection to be added. An explanation regarding local extrema of a function to be given.

Chapter 5 : Differentiation – Geometrical Applications

5.1 Notation for coordinates may be made simpler. We can take $Q = (c + h, f(c+h))$ instead.

Exemplar Material providing alternate treatment to the ones given in the textbook

1. Reference : 1.2.1 (on Limits) of First Year Mathematics - Calculus

Definition of Limit : Given a function f(x), if there exists a number l such that $| f(x) - l |$ can be made as small as we like by making $| x - a |$ sufficiently small, then l is called the limit of f(x) as x tends to a. Then we write $\underset{x \to a}{Lt} f(x) = l$.

This may be put as follows :

The mathematical definition of limit of f(x) as x → a

Corresponding to à n $\in$ > 0 however small, if there exists a $\delta > 0$ such that $|f(x) - l| < \in$ for all n : $| n - a | < \delta$, then l is called the limit of f(x) and tends to a and we write $\underset{x \to a}{Lt} f(x) = l$.

2. *S.T.* $\lim\limits_{x \to -1} (1 - 2x) = 3$

Solution : Putting x = -1 + h.

1 – 2x = 1 – 2 (-1 + h) = 1 + 2 – 2h = 3 – 2h.

As x → 1, h → 0

Hence in the limit,

$$\underset{x \to -1}{Lt} (1 - 2x) = \underset{h \to 0}{Lt} (3 - 2h) = 3 - 0 = 3.$$

3. ***S.T.*** *if* $f(x) = \dfrac{x}{x+1}$ *(x ≠ -1) [Problem No. 20 Art. 1.2.4]*

$$\underset{n \to 1}{Lt} f(x) = \frac{1}{2}$$

Solution: $f(x) = \dfrac{x}{x+1}$. Putting x = 1 + h, as x → 1, h → 0

$$\therefore f(x) = \frac{x}{x+1} = \frac{1+h}{1+h+1} = \frac{1+h}{2+h}.$$

Letting $h \to 0$, $\underset{x \to 1}{Lt} f(x) = \underset{x \to 1}{Lt} \left(\frac{x}{x+1}\right) = \underset{h \to 0}{Lt} \left(\frac{1+h}{2+h}\right) = \frac{1+0}{2+0} = \frac{1}{2}$.

4. *Alternate proof of theorem 1.3.17 (P.No.27)*

Proof: $\underset{x \to a}{Lt} \left(\frac{x^n - a^n}{x - a}\right) = x\, a^{n-1}$

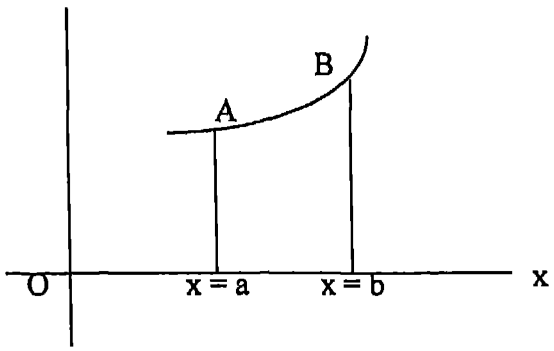
Putting $x = a + h$, as $x \to a$, $h \to 0$, (h sufficiently small)

$\therefore\ x^n = (a+h)^n = a^n + x^{n-1}\, h + \frac{n\,(n-1)}{2}\, a^{n-2}\, h^2$ [By Binomial theorem]

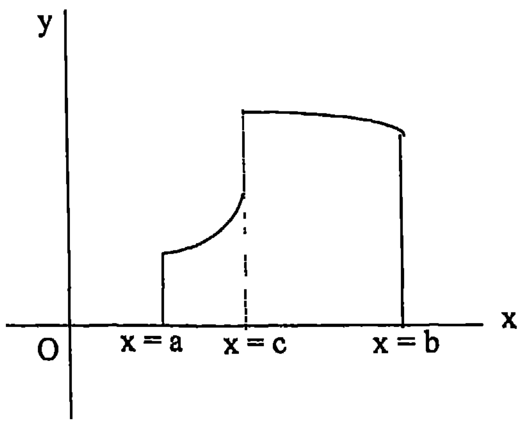
$\therefore\ x^n - a^n = na^{n-1}\, h + \frac{n\,(n-1)}{2}\, a^{n-2}\, h^2 + \ldots\ldots\ldots$

6. Reference : Continuity (P.No.38)
Geometrical Meaning of Continuity/Discontinuity

When the graph of y = f(x) is drawn, the continuity (no break) of the graph means the continuity at any point or over an interval.



Cartesian Graph over [a,b]



Discontinuous Graph at x = c (a < c < b)

7. Alternate Proof of the Chain Rule (Ref. 2.2.8 P.No.51)

Proof: Let y = f(z) and z= g(x). Then y= (f o g) (x) is the composition of f and g.

Denoting the increments in x, z and y by Δx, Δz and Δy respectively,

$$\left.\begin{array}{l}\Delta y = f(z+\Delta z) - f(z) \\ \Delta z = g(x+\Delta x) - g(x)\end{array}\right\}$$

$$\therefore \frac{\Delta y}{\Delta x} = \frac{f(z+\Delta z) - f(z)}{\Delta x} = \frac{f(z+\Delta z) - f(z)}{\Delta z} \times \frac{\Delta z}{\Delta x}$$

$$\therefore \frac{\Delta y}{\Delta x} = \frac{f(z+\Delta z) - f(z)}{\Delta z} = \frac{g(x+\Delta x) - g(x)}{\Delta x}$$

As $\Delta x \to 0$, $\Delta z \to 0$ *and* $\Delta y \to 0$.

$$\therefore \underset{\Delta \to 0}{Lt} \frac{\Delta y}{\Delta x} = \underset{\Delta x \to 0}{Lt} \frac{f(z+\Delta z) - f(z)}{\Delta z} \times \underset{\Delta x \to 0}{Lt} \frac{g(x+\Delta x) - g(x)}{\Delta x}$$

$$= \underset{\Delta z \to 0}{Lt} \frac{f(z+\Delta z) - f(z)}{\Delta z} \times \underset{\Delta x \to 0}{Lt} \frac{g(x+\Delta x) - g(x)}{\Delta x}$$

$$\therefore \frac{dy}{dx} = \frac{df}{dz} \times \frac{dg}{dx}$$

$$\therefore \frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

8. Reference (43,P.No.108) Increasing Decreasing functions

Intuitive ideas (1) Consider a function y = f(x) over [a,b]. As x increases over [a,b],

(a) if f(x) also increases, then f(x) is increasing over [a.b].

(b) if f(x) decreases, then f(x) is decreasing over [a,b].

2. Drawing the graphs of y = f(x) over [a,b].
   (a) when f(x) is increasing, the graph is a *rising graph (fig 1)*
   (b) When f(x) is decreasing ,the graph is a *falling graph* (fig. 2).

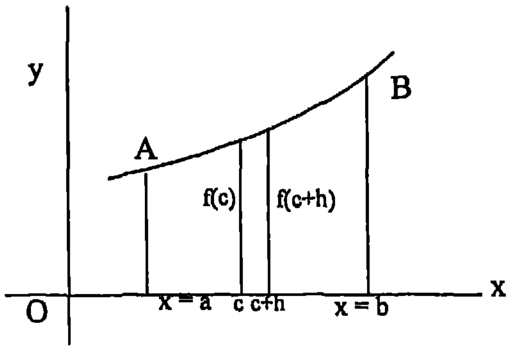


Fig. 1

For a < c < b, (f(c) < f(c+ h), h > 0).

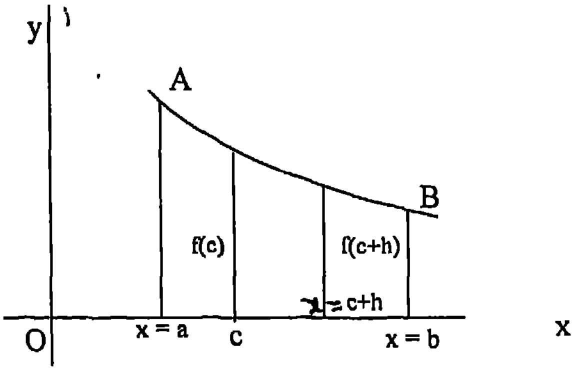


Fig. 2

For a < c < b, (f(c) > f(c + h) , h > 0).

4.4 P.No.113. Maxima and Minima

Definitions:

1. Let y = f(x) be defined over [a,b]. f(x) is said to have a local maximum at x = c ∈ (a,b), if f(c) ≥ f(c – h) and f(c) ≥ f(c + h) , for sufficiently small h. Then f(c) = f_{max} at x = c [See Fig.1].

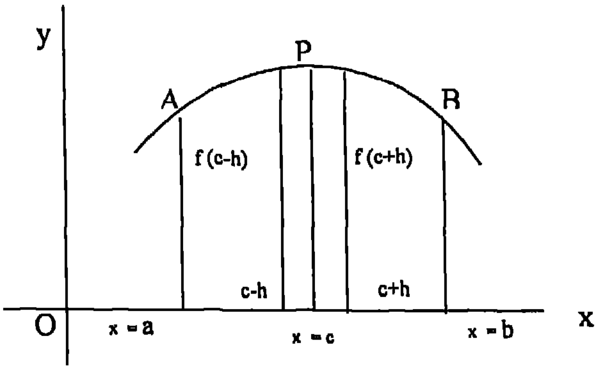
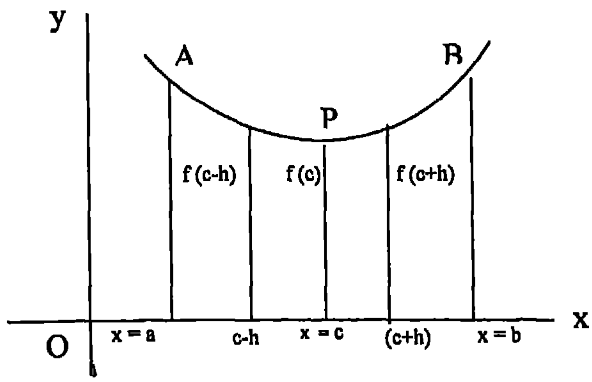


Fig. 1

2. $f(x)$ is said to have a local minimum at $x = c \in (a,b)$, if $f(c) \leq f(c-h)$ and $f(c) \leq f(c+h)$, for sufficiently small h. Then $f(c) = f_{min}$ at $x = c$. [See Fig.2].

f(c-h)



Necessary condition for a maximum or a minimum for a differentiable function:

Theorem 1:

Let $y = f(x)$ be differentiable at $x = c$ and over some interval $(c - h,\ c + h)$, $h > 0$, having a maximum or minimum value at $x = c$. Then $f'(c) = 0$.

Definition: A point $x = c$, such that $f'(c) = 0$ is called a critical point/ stationary point/ turning point of $f(x)$.

Note : The converse of this theorem is *not true* always.

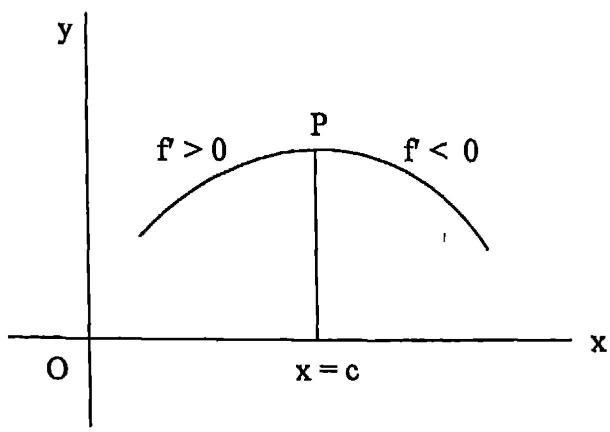
E.g. $f(x)= x^3$, $f'(x) = 3x^2 = 0$ when $x = 0$.

But $f(x)$ is neither maximum nor minimum at $x = 0$.

Sufficient conditions for a Maximum/Minimum:

Theorem 2 :

Let $y = f(x)$ be differentiable at $x = c$ and over some interval $[c - h, c + h]$, $h > 0$.



a) If $f'(x) > 0$, when $x < c$

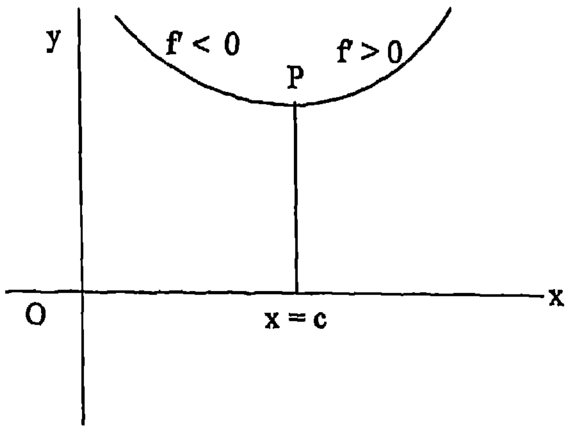
$f'(x) < 0$, when $x > c$

then f(x) has a maximum at $x = c$.

b) If $f'(x) < 0$ when $x < c$

$f'(x) > 0$ when $x > c$

then f(x) has a minimum at $x = c$.



These conditions are used to find the existence of Maximum/minimum of the given function and the test is called the ***First Derivative Test.***

Let f(x) be continuous and different over $[c - h, c + h]$.

Sign of $f'(x)$ when passing through a critical point $x = c$			**Character of the critical point $x = c$.**
$x < c$	**$x = c$**	**$x > c$**	
+	$f'(c) = 0$	-	Maximum at $x = c$.
-	$f'(c) = 0$	+	Minimum at $x = c$.
+	$f'(c) = 0$	+	Neither maximum nor minimum. f(x) is increases.
-	$f'(c) = 0$	-	Neither maximum nor minimum f(x) decreases.

Testing f(x) for maximum or minimum in the second derivative $f''(x)$:

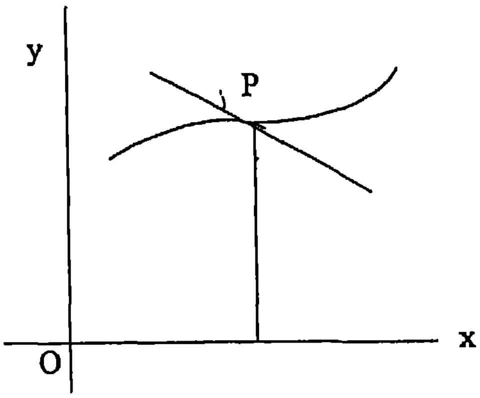
Theorem 3 : Let $f'(c) = 0$. Then f(x) has a maximum or a minimum at $x = c$, according as $f''(c) < 0$ or $f''(c) > 0$. This is known as the ***second derivative test.***

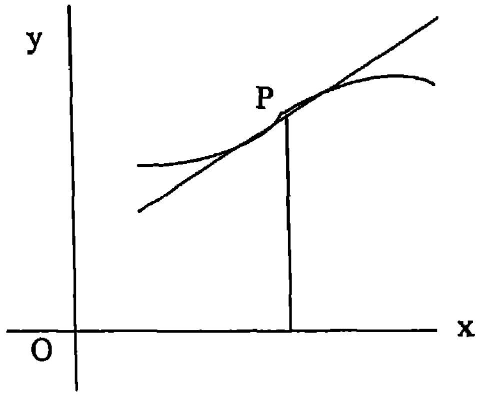
$f'(c)$	**$f''(c)$**	**Characteristic of the critical point $x = c$**
0	-	Maximum point
0	+	Minimum point
0	0	Unknown

When $f''(c) = 0$

Definition: The point $x = c$ which separates the convex part of,a continuous curve $y = f(x)$ from the concave part is called ***a point of inflection.***

Theorem 4 : Let $y = f(x)$ by differentiable over $[c - h, c + h]$, $h > 0$ possessing the second derivative $f''(x)$. If $f''(x)$ changes the sign when passing through $x = c$, while $f''(c) = 0$, then $x = c$ corresponds to a ***point of inflection.***





$f''(c)$	$f''(x)$	**Characteristic of $x = c$**
0	- for $x < c$ + for $x > c$ or + for $x < c$ - for $x > c$	$x = c$ is a point of inflection.

Note: This implies $f''(c) = 0$ and $f'''(c) \neq 0$.

Intermediate First Year Algebra, Trigonometry

Chapter – 1 (Functions/Mappings)

Reference Unit	Page No	Suggestions/Recommendations For Teachers
1.2.5, 1.2.6 to 1.2.17	10-17	Most of the these theorems may be deleted and assertion of some basic theorems may be explained through examples and diagrams.
1.3.6	19	Example of implicit function is not clear. In fact, y as a function of independent variable x is defined by the equation/ relation $x^2 + y^2 - 1 \doteq 0$ where $f(x,y) = x^2 + y^2 - 1$

Chapter – 2 (Surds)

Reference unit	Page No	Suggestions/Recommendations For Teachers
2.1.4 2.21 Problem Ex.2(a)	42 43 52	These are to be deleted.

Chapter – 6 (Binomial Theorem)

Reference Unit	Page No	Suggestions/Recommendations For Teachers
6 1.17	127-128	Proof to be excluded.
4.1.1	79	Only the alternative statement to be retained.
4.1.2 4.1.3	79-80	These are to be deleted.
4.3.3 4 3.8	82	These may be entirely deleted.
Problems 8 & 9	175	These problems may be deleted as no principle of mathematical induction is used in solving these problems.

Chapter – 10 (Complex Number)

Use of ordered pair notation (a,b) for complex numbers should be avoided. Proofs of many theorems on algebra of complex numbers may be avoided. More diagrams or use of Argand plane may be there, wherever feasible. It is better to rewrite this chapter. Some sample materials have been given.

Chapter – 11 (De Moivre's Theorem)

Articles 11.4 and 11.5 - To be deleted.

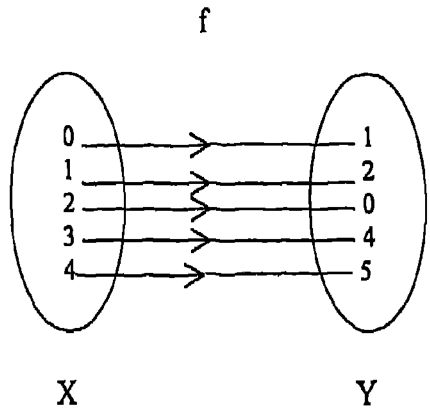
Part B

Illustrative Instructional Materials

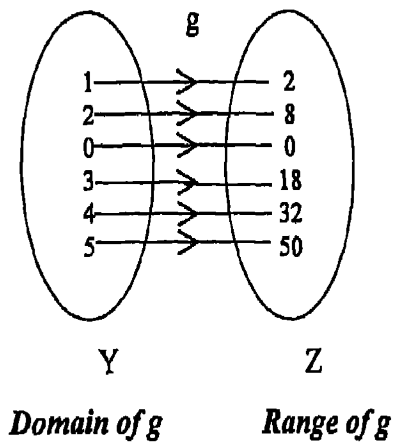
Composition of Functions

Let $f(x) = x + 1$ and the domain of f be $X = \{ 0,1,2,3,4\}$. Let $g(x) = 2x^2$ and the domain of g be $Y = \{0,1,2,3,4,5\}$. We find that the range of f is $\{ f(0), f(1), f(2), f(3), f(4) \}$, that is $\{1,2,3,4,5\}$.

In fact, with some ulterior motive only, we selected the range of f, a set which is a subset of the domain of g.



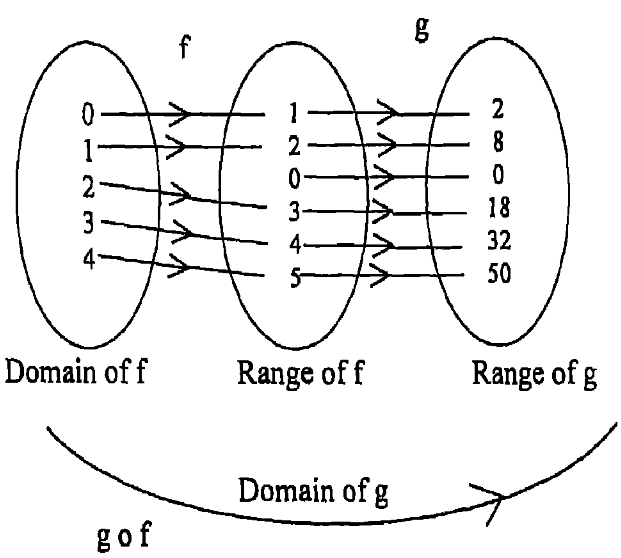
Domain of f *Range of f/Domain of g*



Now let us consider the function $h(x) = 2x^2 + 4x + 2$ with the set X as the domain. Then the range of the function h will be $\{2,8,0,18,32,50\}$. Here it is to be observed that $h(x) = 2x^2 + 4x + 2 = 2(x^2 + 2x + 1) = 2(x + 1)^2 = g(x + 1) = g(f(x))$. This means that $g(f(x)) \equiv 2x^2 + 4x + 2$.

The composite function (or simply, the composition) of the function f and the function g which is symbolically denoted by g o f is a function from X to Z given by g o $f(x) = g(f(x))$ where $x \in X$.

Here it is important to note that the range of f must be a subset of the domain of g for properly defining the composition function g o f.



Ex. 1 : Let X = { 1,2,3, -1}

Y = { 2,3,4,5,6,0}

Z = { 6,7,8,9,10,4}

Let f : x → Y and g, Y → Z be defined by f(x) = x + 1 and g(x) = x + 4 respectively. Describe the composite function g o f and show the same using diagrams.

Solution 1 : g o f is the function h : X → Z where h(x) ≡ (g o f) (x) = g(f(x)) = g(x + 1) = (x + 1) + 4 = x + 5.

So (g o f) (1) = h(1) = 1 + 5 = 6.

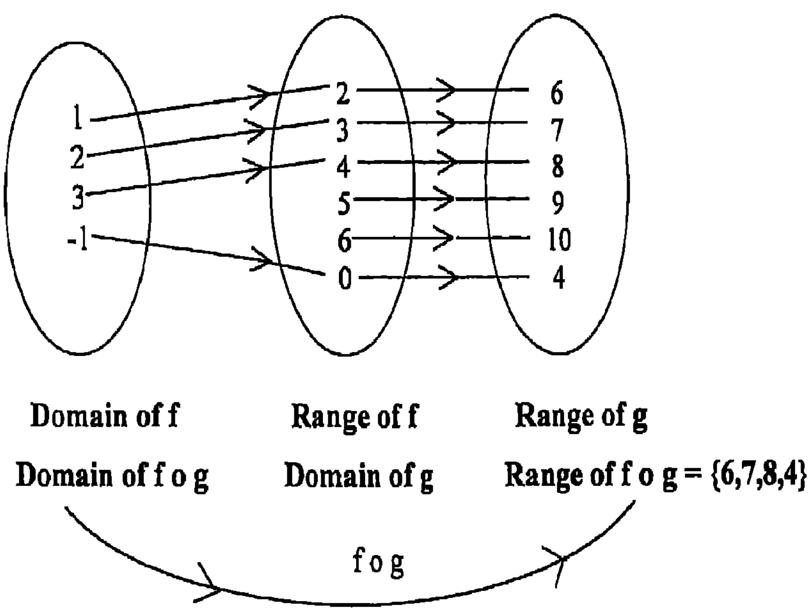
(g o f) (2) = h(2) = 2 + 5 = 7.

(g o f) (3) = h(3) = 3 + 5 = 8.

(g o f) (-1) = h(-1) = -1 + 5 = 4

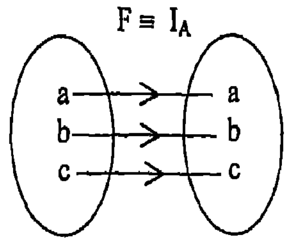
∴ Domain of f o g = x. Range of f o g = {6,7,8,4}

Pictorial Reprentation of g o f



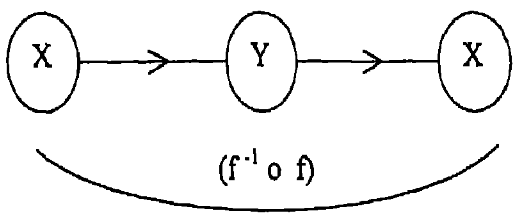
Theorems on Inverse functions, Identify mapping and Composite function :

Consider $f : A \to A$, where $f(x) = x, x \in A$. Pictorially,

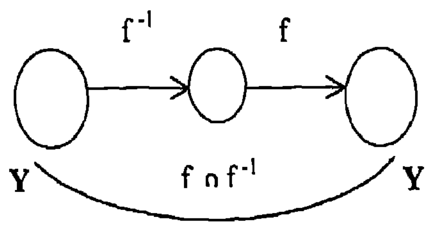


Clearly f is one-one and onto. f is called the identity function (or mapping) on A and symbolically denoted by I_A.

Let a function $f : X \to Y$ have an inverse function $f^{-1} : Y \to X$. Then the following diagram shows that we can form the composite function (f^{1} o f) which maps X into X.



Also the following diagram shows that we can form the composite function (f o f^{-1}) which maps Y into Y. The most important theorems, on inverse functions and composite functions are given below without proof.



Theorem 1 : If the function $f : X \to Y$ is one-one and onto, i.e., the inverse function $f^{-1}:Y \to X$ exists, then the composite function $(f^{-1} \circ f) : X \to X$ is I_X, the identity function on X and the composite function $(f \circ f^{-1}) : Y \to Y$ is I_Y, the identity function on Y.

Theorem 2 : Let $f : X \to Y$ and $g : Y \to X$ be two functions. Then g is the inverse function of the function f i.e., $g = f^{-1}$ if the composite function $(g \circ f) : X \to X$ is I_X, the identity function on X and $(f \circ g) : Y \to Y$ is I_Y, the identity function on Y.

In theorem 2, both the conditions i.e. (i) $(g \circ f) : X \to X$ is I_X and (ii) $(f \circ g) : Y \to Y$ is I_Y are necessary. This will be evident from the example given below.

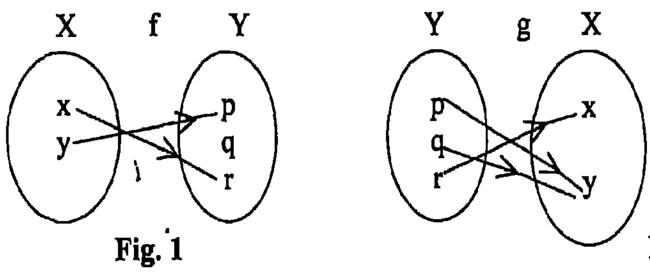


Fig. 1 Fig.2

Let X = {x,y} and Y = {p,q,r}. Define a function $f : X \to X$ as in Fig.1 above and also define another function $g : Y \to X$ as shown in fig. 2 above. We compute $(g \circ f) : X \to X$.

$(g \circ f)(x) = g(f(x)) = g(r) = x$

and $(g \circ f)(y) = g(f(y)) = g(p) = y$

Therefore the composite function $(g \circ f) = I_X$, the identity function on X. But $g \neq f^1$ as the composite function $(f \circ g) \neq I_Y$, the identity function on Y, f not being an onto function.

Chapters 10

Complex Numbers

As remarked earlier Chapters 10 is to be rewritten so that students may be able to correlate the algebraic components of complex numbers with the corresponding geometrical counterparts. Here as exemplary material the following will be dealt with –

a) Definition of complex number and its representation – Argand plane
b) Addition of complex numbers
c) Multiplication of complex numbers
d) Conjugate complex numbers

The definition of Complex Number

We know that the equation $x^2 = -1$ has no real roots (x^2 being never negative). This suggested inventing an imaginary number 2', satisfying $i^2 = -1$ and otherwise satisfying the ordinary laws of algebra. So an expression of the form x + iy is taken to represent the element called the complex number. Here we should note x and y are real and $i^2 = -1$. By the ordinary laws of algebra, we have

$$(x + yi) \pm (x' + y'I) = (x \pm x') + (y \pm y')\, i \qquad (1)$$

$$(x + yi)\ \ (x' + y'\, i)\ = xx' + (xy' + yx')i\ + yy'\, i^2 \qquad (A)$$

As $i^2 = -1$(A) can be written as

$$(x + yi)\,(x' + y'I) = (xx' - yy') + (xy' + y'x)\, i \qquad (2)$$

Definition : A "complex number" is a number of the form x + iy (x,y real, $i^2 = -1$), x and y being called the real and imaginary part of (x,y) respectively. Complex numbers are added and multiplied by the rules :

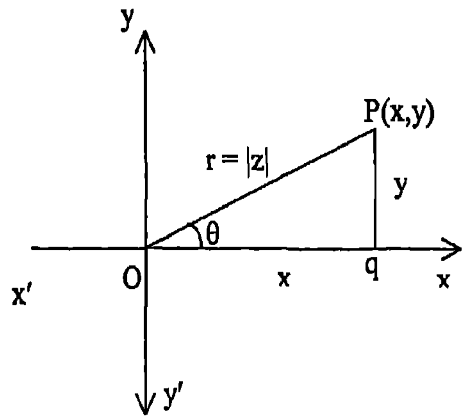
$$(x,y) + i\,(x', y')\ = (x + x') + (y + y')\, i$$

$$(x,y)\,.\,(y, y') = (xx' - yy') + (xy' + yx')\, i$$

The set of complex numbers so defined is generally denoted by C.

Argand Plane

There is a fundamental one-one mapping of the complex numbers onto the points of a Cartesian plane, which is here also called Argand plane. Each complex number (x,y) or z = x + iy corresponds to the point P(x,y) of the Cartesian or Argand plane. Here x and y are the real and imaginary parts of the complex number Z. Note x and y are the abscissa and the ordinate respectively of the point P in the Argand plane. Sometimes the Argand plane is also called the complex plane.



In Argand plane x o x', the x-axis is called the real axis and the y-axis is called the imaginary axis.

Let PQ ⊥ OX, and ∠QOP = θ measured counterclockwise. Now OQ = x, QP = y.

$OP = \sqrt{OQ^2 + QP^2} = \sqrt{x^2 + y^2}$.

Here θ and OP are called the argument (or amplitude) and the modulus of the complex number Z = x + iy. OP is denoted by | Z | or r. Note that $|Z| = r \geq 0$.

Now it is clear that if Z = x + iy, then

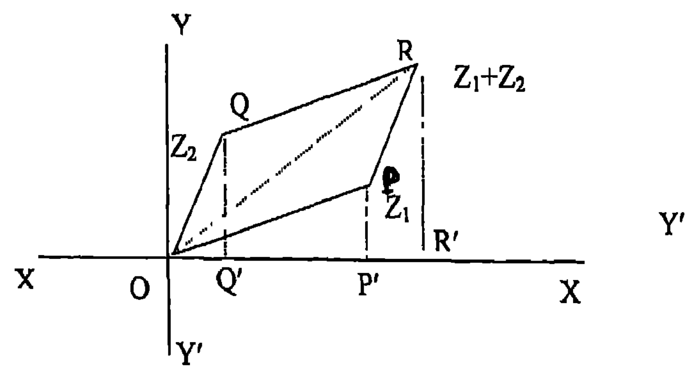
$|Z| = \sqrt{x^2 + y^2}$

$x = r\cos\theta,\ y = r\sin\theta$.

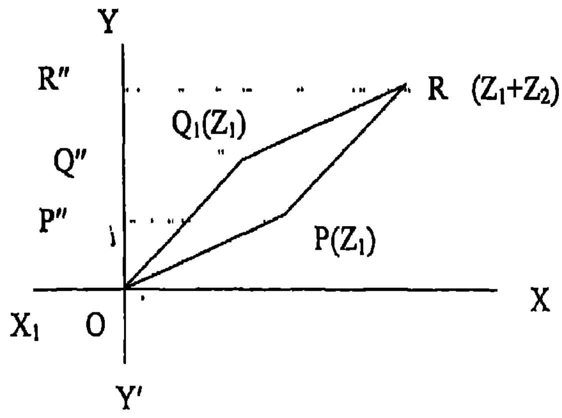
Also as, $\tan\theta = \frac{y}{x}$, ∴ we have $\theta = \tan^{-1}\frac{y}{x}$.

So, if Z = x + iy, we have $Z = r(\cos\theta + i\sin\theta)$, where r and θ are modulus and argument of Z respectively. This form of writing the complex number is called the *polar form*

Addition of 2 complex numbers.



In the above figure P and Q represent the complex numbers $Z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ and OPRQ is a parallelogram. Now QQ′, PP′, RR′ are all perpendicular to Ox, $OQ' = x_2$, $OP' = x_1$, $P'R' = OQ' = x_1$. So $OR' = OP' + P'R' = x_1 + x_2$.



As before we can show in the above figure if $OP'' = y_1$, $OQ'' = y_2$, then $OR'' = y_1 + y_2$.

As $OR' = x_1$ and x_2, $OR'' = y_1 + y_2$. We conclude in the parallelogram OPRQ, if OP and OQ denote $Z_1 = x_1 + iy_1$, $Z_2 = x_2 + y_2 i$ respectively, then OR will represent

$(x_1 + x_2) + (y_1 + y_2) I = (x_1 + iy_1) + (x_2 + iy_2) = z_1 + z_2.$

Product of two complex numbers :

Let $Z_1 = x1 + iy_1 = r_1 (\cos 0_1 + i \sin \theta_1)$

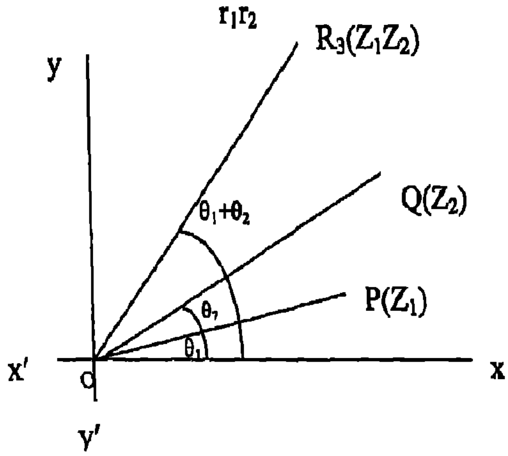
and $Z_2 = x_2 + iy_2 = r_2 (\cos \theta_2 + i \sin \theta_2)$ be two complex numbers.

Then $Z_1 Z_2 = r_1 r_2 (\cos \theta_1 + i \sin \theta_1)(\cos \theta_2 + i \sin \theta_2)$

$= r_1 r_2 [(\cos \theta_1 . \cos \theta_2 - \sin \theta_1 . \sin \theta_2) + i (\sin \theta_1 . \cos \theta_2 + \cos \theta_1 . \sin \theta_2)]$

$= r_1 r_2 [\cos (\theta_1 + \theta_2) + i \sin (\theta_1 + \theta_2)].$

This means that multiplying a complex number Z_1 by another complex number Z_2 results in rotating.



Corollary 1 : If $Z_1 = kZ_2$ where Z_1 and Z_2 are complex numbers and k is real, then amplitude of Z_1 = amplitude of Z_2.

Proof : Let $Z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $Z_2(\cos\theta_2 + i\sin\theta_2)i$. Then $Z_1 = k Z_2$.

$\Rightarrow \quad \theta_1$ = amplitude of $k + \theta_2$

But amplitude of k = 0, k being real.

So $\theta_1 = 0 + \theta_2 = \theta_2$

i.e. the amplitude of Z_1 = the amplitude of Z_2.

Corollary 2 : If amplitude of Z_1 = amplitude of Z_2, then $Z_1 = k Z_2$ (k real).

Proof : As amp. Z_1 = amp. $Z_2 = \theta$ (say) we can write,

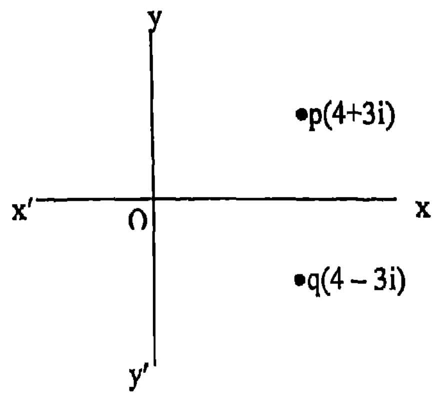
$Z_1 = r_1(\cos\theta + i\sin\theta)$, $Z_2 = r_2(\cos\theta + i\sin\theta)$.

So, $\dfrac{Z_1}{Z_2} = \dfrac{r_1}{r_2} = k$ (say), a real number

$\Rightarrow Z_1 = k Z_2$.

Conjugate Complex Numbers

Definition· x + iy and x – iy are said to be ***conjugate*** of each other. That is, x + iy is a ***complex conjugate*** of x – iy and vice versa. Geometrically x + iy and x – iy are symmetrically situated about the x-axis of Argand plane.



In the above figure, the complex numbers $4 + 3i$ and $4 - 3i$ are denoted by points P and Q which are situated symmetrically about x-axis.

Intermediate First Year

Permutations and Combinations

5.0 The introduction is brief and convincing. But the teacher, giving several examples like considering a group of boys from the class itself will make the students get interest in the topic.

5.0.1 The fundamental principle is well explained through 5.0.3 and 5.0.4 (fundamental principle of counting that if one work can be done in m ways and another in n ways, then both of them together can be done in $m \times n$ ways).

5.1 The definitions of different types of permutations are clear but the teacher should devote more time in making them clear in the minds of the students.

5.1.4 The notation of permutation ${}^nP_r$ and in 5.1.5, the notation of factorials ⊓n (0 = 1 defined) should be given more time by taking smaller numerical examples. For example, if we take n = 5 and r = 2, then ${}^nP_r = {}^5P_2$ i.e. taking 2 things out of 5 things at a time. In the two places given below the first

5 4

place can be filled in 5 ways and the second place can be filled in four ways. Therefore, from the fundamental principle, both of them can be filled in 5×4 ways.

i e. ${}^5P_2 = 5 \times 4$

$$= \frac{5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1}$$

$$= \frac{5}{3}$$

$$= \frac{5}{5-2}$$ which leads to ${}^nP_r = \frac{n}{n-r}$ and $5 = 5 \times 4 \times 3 \times 2 \times 1$

which leads to n = n(n-1) (n-2).......1.

Several such examples will increase the grip of a student in understanding the theorem ${}^nP_r = \frac{n}{n-r}$ and the dealing of cancellations in factorials. For

example, a student immediately may not be able to cancel $\frac{n-1}{n-3} = (n-1)(n-2)$ but if we just use a small numerical in place of n = 7,

we get $\frac{7-1}{7-3} = \frac{6}{4}$

$= \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1}$

$= 6 \times 5$

$= (7-1)(7-2)$

which in turn clarifies $\frac{n-1}{n-3} = (n-1)(n-2)$.

Similarly, in the case of say $\frac{n-r}{n-r-3}$, if small numerical values say n = 7, r = 2,

then $\frac{7-2}{7-2-3} = \frac{5}{2}$

$= \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1}$

$= (7-2)\ (7-3)(7-2)$

$= \{7-2\}\{7-(2+1)\}\{7-(2+2)\}$

which in turn gives

$$\frac{n-r}{n-r-3} = (n-r)(n-r-1)(n-r-2)$$

Worked out examples (5.1.11 to 5.1.22) is exhaustive but regarding the problem of rearrangement of word (like 5.1.17), if the teacher gives an idea of the mechanism of arrangement of words in a 'dictionary' to the students, it will add to their understanding more. Such problems require elaborate explanation but at the same time, precise steps as given in the book will save the time of the student in the examinations.

5.1.23 Theorem $^nP_r = {}^{(n-1)}P_r + r^{(n-1)}\ P_{(r-1)}$ (the magnification of (n – 1) in the textbook may mislead a student) understood by the student by both the proofs will enhance their comprehension.

5.1.24 Circular permutations are well explained as also Theorem 5.1.27.

5.1.25 5.1.35 Theorem of m identified objects out of n $(m < n)$ is well explained for the result $\frac{n}{m}$ and the note 5.1.36 of the result $\frac{n}{m\ p\ r\ t}$ is important and could be explained one or two steps further as $\frac{n}{m}\Big/p = \frac{n}{m\ p}$ and $\frac{n}{m\ p}\Big/r = \frac{n}{m\ p\ r}$ .... etc. Worked out examples are suitable and veritable

Combinations

Definition (5.2.1) and Proof (5.2 3) are neat. They should be made interesting by the teacher by taking small numerical values and live examples like selection of students in the class for a games team, etc.

5.2.5 Theorem ${}^{n}C_{r} = {}^{n}C_{n-r}$ is an interesting result very useful in jugglery of ${}^{n}C_{r}$ *to* ${}^{n}C_{n-r}$ in problems. Hence, its importance should be pointed out in the beginning thoroughly so that the students will be at ease using it.

Theorem 5..27, i.e. if ${}^{n}C_{r} = {}^{n}C_{s}$, then either r = s or r + s = n is interesting.

Theorem 5.2.8 is just Theorem 5 2.9 with (n – 1) in place of n. ***Hence 5.2.8 is redundant.*** Theorem 5.2.9 i.e. ${}^{n}C_{r} + {}^{n}C_{r-1} = {}^{(n+1)}C_{r}$ should also be understood both ways by the students to increase their comprehension. Corollaries 5.2.11 to 5.2.17 are very important in increasing the sharpness of a student's mind; they should be dealt with very well in the class.

A good variety of examples are given. The following exercises 5(c) and 5(d) have good number of problems.

On the whole, the chapter on Permutations and Combinations is very important and adds to the creativity of students. Time devoted in making it interesting to the students and making them have a grasp over it is ***worth.***

Intermediate Second Year

Vector Algebra

Reference	Recommendations
Ch. 4. 4.1.6 (P.No.3)	In the definition of *free vectors*, 'vectors under equivalence character are called free vectors', it is not clear what 'equivalence character' means. This needs to be clarified.
4.3.2 (P.No.6)	This ay be deleted. The triangle is true for 'any triangle'. The figure (of art. ⌞led Δle) is misleading. Triangle law may be stated in general terms verbally.
4.3.7 (P.No.11)	Why take the vectors $\vec{a}$ *and* $\vec{b}$ are perpendicular vectors? The figure is misleading.
4.3.10 (P.No.12,13)	The triangle Inequality $\mid \vec{a} + \vec{b} \mid \leq \mid \vec{a} \mid + \mid \vec{b} \mid$ is an important theorem and the theorem is just a restatement of the theorem in geometry – In a triangle, the sum of any two sides exceeds the third side. Hence proof of this theorem is no proof at all. This can be deleted and only the statement of the inequality can be retained.
4.4.3 Theorem. (P.No. 16 & 17)	Only statements may be retained and the elaborate proofs may be omitted
4.5 Angle between two vectors (P.No.19 & 20)	The entire matter may be omitted.
4.5.4 (P.No.20 & 21)	The theorem on division formula and proof is very elaborate. It can be made briefer.
4.5.9 (P.No.23 & 24)	Mere statement of the collinearity condition is enough.
4.7 (P.No.32 & 33)	The matter on basis .... and RH/LH systems are unnecessary. It is better to directly discuss *I, j, k* vectors and vectors in R_3. Explanations regarding RH/LH not clear. It would be better to explain RH/LH with the help of Right hand/Left hand Screw Rule.
4.7.8 (P.No.34)	Define the d.c's merely as cos α, cos β, cos γ where α, β, γ are the triangles made by $\bar{\lambda}$ with the unit vectors respectively.
4.8 (P.No.41)	Line and plane in R^3 may be deleted. This adds to the burden unnecessarily.

Overall recommendations on Chapter 4 :

1. The volume of the inclined matter may be reduced considerably (atleast by 30%).
2. Notations may be simplified.
3. Concepts of 3-D Geometry as line/plane may be removed.

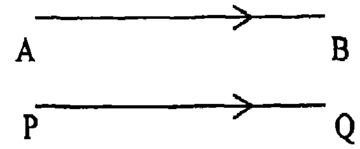
Chapter 5 Multiplication of vectors

Reference	Recommendations
5.1.1 (P.No.53)	Definition of $\vec{a} \cdot \vec{b}$ may be made simpler by avoiding difficult notations and using diagrams.
5.1.7 (P.No.55)	The definition is confusing (due to the difficult notations). It can be made simpler.
5.1.9 (P.No.56)	Proof of the theorem may be deleted. Only statement of the theorem may be retained.
5.2.2 (P.No.57)	Only statements of the theorem may be retained.
5 2.4 (P.No.59)	Proof of the theorem may be simplified Notations are difficult and add to the students' difficulty in understanding.
5.3.1 (P.No.60)	Give the information as a multiplication table. <table><tr><td>×</td><td>i</td><td>j</td><td>k</td></tr><tr><td>i</td><td>1</td><td>0</td><td>0</td></tr><tr><td>j</td><td>0</td><td>1</td><td>0</td></tr><tr><td>k</td><td>0</td><td>0</td><td>1</td></tr></table>
5.3.7 (P.No.62-63)	These may as well be given under exercises. These need not be taken as results to be remembered by the students.
5.6.3 (P.No.79)	Only statements in the theorem may be retained.
5.6.4 (P.No.80-83)	The proof of the theorem (on Distributive law) may be replaced by a simpler proof taking $\vec{a}, \vec{b}, \vec{c}$ as vectors in R^3 in the form $xi+yj+zk$. *The given proof is very cumbersome, unclear and lengthy*
5 9.10 P.No 101	Proof of the theorem can be simplified. As it is, the proof is cumbersome.

1. **Exemplar Material on Vectors**
   Definition of free vectors/bound (localized) vectors

A vector which is determined by its direction and magnitude but not by the point of application or line of action (or support) is called *a free vector*. A vector which is not a free vector is called a *bound vector.*

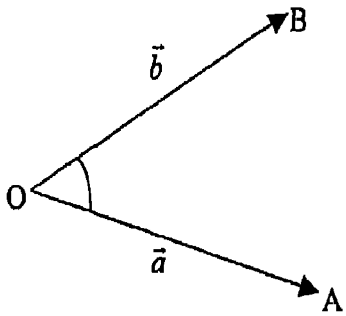
In the case of free vectors, vectors are represented by directed line segments of same direction and equal lengths are equal.
Equivalently, a vector does not alter if it is moved parallel to itself (without rotation). This gives us the freedom to draw a few vector's diagram anywhere.



If AB = PQ, then $\overrightarrow{AB} = \overrightarrow{PQ}$.

2. Definition of scalar (or dot) product

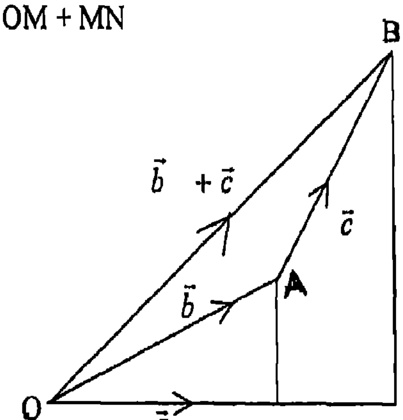
$\vec{a} \bullet \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta$, θ being the angle between $\vec{a}$ **and** $\vec{b}$.



3. Proof of Distributive Law

$$\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$$

In the figure, ON = OM + MN



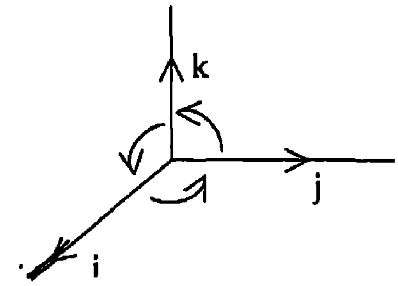
$\because$ Projection of $\overrightarrow{OB}$ on $\vec{a}$ = Projection of $\vec{b}$ on $\vec{a}$ + projection of $\vec{c}$ on $\vec{a}$.

$$\therefore \quad \frac{\vec{a} \cdot (\vec{b} + \vec{c})}{|\vec{a}|} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} + \frac{\vec{a} \cdot \vec{c}}{|\vec{a}|}$$

$$\therefore \quad \vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$$

4. **Multiplication (×) Table i,j,k.**

×	i	j	k
i	0	k	-j
j	-k	0	i
k	J	-i	0



5. **Proof of Distributive Property**

$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$

Taking $\vec{a} = a_1\ i + a_2\ j + a_3\ k$

$\vec{b} = b_1\ i + b_2\ j + b_3\ k$

and $\vec{c} = c_1\ i + c_2\ j + c_3\ k$

$$\vec{a} \times (\vec{b} + \vec{c}) = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 + c_1 & b_2 + c_2 & b_3 + c_3 \end{vmatrix}$$

$$= \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} + \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$= \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$

Intermediate Second Year

Probability

As the chapter of Probability is most useful in real life situations, it must be dealt with the students in a way such that both the concepts and the problems seep into their minds very well and in that process, the students' sharpness increases with respect to any real life situation.

6.1 Random Experiments and Events which are nothing but regular features and activities of our life which can be understood better with more of mathematical insight.

Tossing of a coin, rolling a dice or understanding random experiments with a pack of cards, or with a race of horses is alright. *But* considering the attendance of a number of students in a class – say for one month and on that basis informing each student of his attendance *as a probability* will certainly make the student more conscious about his attendance and also creating more awareness about Probability. Similarly, observing the colours of shirts the students wear, we can one day say that green colour is very popular among students, quoting our observations Heights of the students, weights of the students, number of a particular brand of cars on the road, number of spiders killed when we clear the cob-web etc. Hundreds of intelligent examples can be imagined and put before the students in a colourful manner which will create Probability Consciousness in the students.

6.2 Classical Definition of Probability

Definition goes very well

Probability of a particular event = $P(E) = \frac{m}{n}$ could be explained by means of colourful examples like the probability of a good song being played when a number of songs are given to the group to play any song any number of times limiting the number of playing, say 50 times.

The complementary probability P(E) and P(E) + P(E) = 1 etc. could be explained by means of a variety of interesting examples and thus create a thorough understanding will make the student understand the problems easily. Sample spaced, Impossible event, Certain event, Mutually exclusive events, Exhaustive Events, Complementary events, Probability function (all these topics are dealt well).

6.2.10 Addition Theorem is dealt well. Worked examples are also exhaustive. The concerned lecturer must ensure that the students go through the solved examples (as such for any topic) without fail as they will be able to learn the steps correctly without wasting time. In the class, while explaining a problem, the teacher may not write the steps properly.

6.3 Conditional Probability, Multiplication theorem (with respect to independent probabilities), Bay's theorem are dealt well. But Bay's theorem could be a hard spot unless thoroughly studied by the teacher.

7. ***Random Variable and Distributions***

7.1.1 Random variable and 7.1.2 Probability Distribution function are very clearly explained with examples.

7.1 5 Discrete Random Variable and 7.1.6 Mean and Variance of a variable is defined clearly and a suitable example given.

Theorem 7.1.8 i.e. $\sigma^2 + \mu^2 = \sum x_n^2 \, p_n$ is very clearly explained. A good example also follows.

7.2 Theoretical Discrete Distributions:

7.2.1 Binomial Distribution and its variants *n and p* are dealt with clearly and suitable examples given. 7.2.7 Poisson Distribution and 7 2.8 Poisson Variate is also dealt well. Examples given in the exercises 7(b) and 7(c) contain a good number of real life problems.

Intermediate Second Year
Co-ordinate Geometry

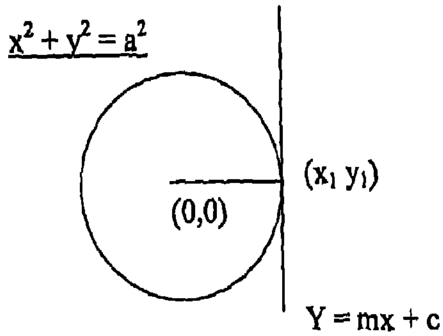
Reference Unit	Page No.	Article	Recommendations
1.1.3	2	To find the conditions for the second degree equation to represent a circle	Instead, it is enough to show that the equation $x^2+y^2+2gx+2fy+c=0$ represents a circle which is much simple. Remarks: Worked out in the latter pages.
1.1.15	5	To find the equation of the circle with A (x_1, y_1) & B $(x_2 y_2)$ as ends of a diameter	Alternate proof may be given is using angle in a semi circle is a right angle and also that the product of the slopes is equal to –1, taking $P(x, y)$ any point on the circle.
1.1.16	6	To show that the angle in a semi circle is 90^0	Not necessary to have this theorem here. Same will be learnt the students in pure geometry in the Secondary School itself.
1.2.5	13	General discussion of the intersection of a line and a circle.	Instead, it is better to find the condition for the line $y=mx+c$ to be a tangent to the circle $x^2+y^2=a^2$ and also the point of contact simultaneously. This book work should come after the equation of the tangent at a given point to a given circle.
1.3.3	25	Equation of the tangent to the circle at the point $(x_1 y_1)$	Alternate proof may be given i.e. by finding the equation of the tangent using point,slope form of the equation to a straight line. Slope will be found out by calculus method.
1.4	27	Pole and Polar	This unit may be removed from the syllabus. It is bit above the standard of average students of +2 course.
1 4.17 & 1 4 19	33 34	Theorems on Inverse points and secant	These theorems and problems based on it may be removed from the syllabus. Because it adds to the heavy syllabus which is not necessary.
2.1.2	52	Angle between two circles	It is nothing but angle between two curves which is in calculus and hence not necessary here. Therefore, in particular, Orthogonal Circles will be sufficient.
3.3.1	112	Chord of contact-pole and polar	This is above the standard of +2 students and hence may be removed.

POLAR COORDINATES

6.1		Distance formula, Area of a Δ^{le}, equation to a straight line etc., in polar co-ordinates.	These are studied usually in first year in Cartesian coordinates. Hence, it is better to include the same in polar coordinates in first year syllabus. Teachers may also find this easier to teach.

Worked out Sheets

Unit of Ref.	
1.1.3	To show that $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle. $x^2 + y^2 + 2gx + 2fy + c = 0$ $(x^2 + 2gx) + (y^2 + 2fy) = -c.$ $\Rightarrow (x + g)^2 + (y + f)^2 = g^2 + f^2 - c$ $\Rightarrow [x - (-g)]^2 + [y - (-f)]^2 = (\sqrt{g^2 + f^2 - c})^2.$ This is of the form $(x - \alpha)^2 + (y - \beta)^2 - r^2$. $\therefore$ center $(\alpha, \beta) = (-g, -f)$ Radius $= r = \sqrt{g^2 + f^2 - c}$.
1.2.5	Condition for the line $y = mx + c$ be a tangent to the circle $x^2 + y^2 = a^2$ and to find the coordinates of the point of contact. Ans: Equation of the tangent to the circle $x^2 + y^2 = a^2$ at $(x_1 y_1)$ is $xx_1 + yy_1 = a^2$ $\Rightarrow x_1x + y_1y - a^2 = 0$ .... (1) The line $y = mx + c$ can be written as $mx - y + c = 0$ .... (2) (1) and (2) represents the same, therefore, coefficients of the corresponding terms are proportional. $\frac{x_1}{m} = \frac{y_1}{-1} = -\frac{a^2}{c}$ $\Rightarrow x_1 = -\frac{a^2m}{c},\ y_1 = \frac{a^2}{c}.$



$(x_1\ y_1)$ is a point on the circle $x^2 + y^2 = a^2$.

$$\therefore \left(-\frac{a^2 m}{c}\right)^2 + \left(\frac{a^2}{c}\right)^2 = a^2$$

$$\Rightarrow \frac{a^4 m^2}{c^2} + \frac{a^4}{c^2} = a^2$$

$\Rightarrow a^2m^2 + a^2 = c^2$

$\Rightarrow a^2 \ (1 + m^2) = c^2$

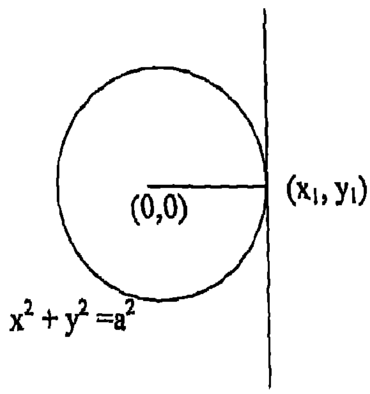
$\Rightarrow c = \pm a \sqrt{1 + m^2}$ is the required condition.

$\therefore$ Equation of the tangent is $y = mx \pm a \sqrt{1 + m^2}$. Point of contact. $\left(\frac{-a^2m}{c}, \frac{a^2}{c}\right)$

1.3.3 Equation of the tangent to the circle $x^2+y^2=a^2$ at $(x_1\ y_1)$

$x^2+y^2=a^2$

Differentiating with reference to x



$$2x+2y\frac{-dy}{dx}=0$$

$$\frac{dy}{dx}=\frac{-x}{y}$$

$$\left.\frac{dy}{dx}\right)_{x_1 y_1}=\frac{-x_1}{y_1}$$ =slope of the tangent at (x_1, y_1)

$\therefore$ Equation of the tangent line in the point-slope form of the equation to a straight line is

$y-y_1=m(x-x_1)$

$$\Rightarrow y-y_1=\frac{-x_1}{y_1}(x-x_1)$$

$$\Rightarrow yy_1-y_1^2=-xx_1+x_1^2$$

$$\Rightarrow x_1^2+y_1^2=xx_1+yy_1$$

(x_1y_1) is a point on the circle $\therefore$ $x^2+y^2=a^2$

$\therefore$ we get $a^2=xx_1+yy_1$

$\Rightarrow xx_1+yy_1-a^2=0$ which is the required equation.

Reference Unit	Page No	Article		Recommendations
		Chapter 1 : Partial Differentiation	1.	This unit is unnecessary since it is not used anywhere else in the book (in other units). Removing this unit makes the content lighter. However, if it is strongly felt to retain it, suitable modifications to make it lighter may be made. Continuity in the content is not lost by deleting the entire chapter. Proofs of theorems (2.3.1, 2.3.2) may be deleted.
2.8	32-33	Chapter 2 : Methods of Integration – Integration by parts	2.	Here to help the students to make the correct choice of the 'first function' while integrating the product of two functions using the Rule – $\int 1 \times 2 = 1 \times \int 2 - \int \int 2 \times 1'$ 1st fn, 2nd fn, the "I LATE" rule may be given
	36		3.	The working rule for $\int \frac{dx}{\sqrt{ax^2 \pm bx + c}}$, $\int \sqrt{ax^2 + bx + cdx}$ can be suggestive as – reduce it to one of the forms $\int \frac{dx}{\sqrt{a^2 \pm x^2}}$, $\int \frac{dx}{\sqrt{x^2 - a^2}}$, $\int \sqrt{a^2 \pm x^2}\, dx$, $\int \sqrt{x^2 - a^2}\, dx$ by completing the squares.
			4.	For integrals (xiii), (xiv), (xv), a substitution on putting $px + q = \lambda\,(2ax + b) + \mu$ is better.
2.10	42,43			The process of resolving into partial fractions is taught earlier (First year – Chapter 7). Hence the articles 2.10.1 to 2.10.4 ***may be removed.***

Refe-rence Unit	Page No	Article	Recommendations
		Chapter 3 : Definite integrals	Definitions 3.1.1 to 3.1.3 are unnecessary as they are not used elsewhere in the chapter (since the Fundamental theorem of Integral Calculus is stated without the proof). It is necessary to link the definition of definite integral with the definition of Indefinite Integral as anti-derivative. Instead, the conclusion of the theorem serves as the definition of $\int_a^b f(x)\,dx = F(b) - F(a)$ when $F'(x) = f(x)$.
3.2.6 Theorem	75	Formula	The formula for $\int_0^{\pi/2} \sin^m x \cos^n x\,dx = I_{m,n}$ may be made shorter as $I_{m,n} = \frac{(m-1)(m-3)\ldots(n-1)(n-3)..}{(m+n))(m+n-2)\ldots.} \times K$ where $K = \begin{cases} \pi/2, \text{ if } m, n \text{ are both even} \\ 1, \text{ otherwise} \end{cases}$
		Chapter 4 · Numerical Integration	The unit 4.1 on area under curve may be *annexed to the* previous chapter.

Definite Integral and Indefinite Integral –An Explanation

Even though we introduce Indefinite Integral of a function on the anti-derivative of another function, for pedagogic reasons,

[f(x) dx = F(x)+ c when F′(x) = f(x)], it is logical necessity to explain how this follows from the definition of definite integral. Here the fundamental theorem(s) of Integral Calculus gains importance.

Theorem 1 : If f(x) is a continuous fuction and $F(x) = \int_a^x f(t)\,dt$,

then F′(y) = f(x).

Theorem 2: If F(x) is an antiderivative of a continuous function f(x)

(i.e. F'(x)=f(x)), then $\int_a^b f(x)\ dx = F(b) - F(a)$.

This formula is called the Newton-Leibnitz formula.

From theorem 1, F(x) = $\int f(x)\ dx$ = Antiderivative of f(x) follows. This is the justification for taking $\int f(x)\ dx = F(x) + c$ when f(x) = F′(x).

Questionnaire for the identification of hard spots in Mathematics at Intermediate/ +2 level

1. Name of the teacher :

2. Name of the school/college presently working with address. :

3. Number of years Experience in Teaching Mathematics :
   a) At Secondary Level :
   b) At Intermediate/+2 Level :

4. List of topics in Mathematics at +2 level is enclosed Please read each topic and determine the extent of difficulty felt by you in teaching the same to students of respective classes. Your responses may kindly be indicated by encircling the serial number on a 5-point scale against each subtopics/sections/subsections.

   1 = Very easy
   2 = Easy
   3 = Some what difficulty
   4 = Difficulty
   5 = Very difficulty

   and also suggest the measures to be taken to over come the difficulties in teaching the topics identified by you as difficult and very difficult.

MATHEMATICS-1A
Intermediate 1-Year Course
ALGEBRA

Sl.No.	Topics/Chapter	Sub-topics/Sections/Sub-sections						
1	Functions or Mappings	1.1	Definitions of one one, onto, Bijection functions, identify And constant functions Equality of two functions.	1	2	3	4	5
		1.2	Definition of Inverse function, Composites function and inverse of composite function.	1	2	3	4	5
		1.21	FA B g: B C and bijection, then gof: A C is also bijection	1	2	3	4	5
		1.22	Let A & B be two sets, if f: A B is A bijection then f: B-A is also a Bijection.	1	2	3	4	5
		1.23	If f: A-B and g: B-C are two bijective Functions the (gof)=fog"	1	2	3	4	5
		1.24	If f:A-B and g B A are two functions such that gof=1 and Fog=1 then g=f	1	2	3	4	5
		1.25	If f: A B g: B C and h: C D are any 3 Functions then ho(gof)=(hog) of	1	2	3	4	5
		1.2	Definition of real valued function or R, Domain & Range, Algebra of real valued Functions.	1	2	3	4	5
2	Surds	2.1	Extracting Square roof of a surd. 2 periods	1	2	3	4	5
		2.2	Cube roof of a surd	1	2	3	4	5
		2.3	Rationalising factors (upto 3^{rd} degree) 2 periods	1	2	3	4	5
3	Logarithms	3.1	Definitions Introduction of common logarithms.	1	2	3	4	5
		3.11	log mn=log m+log n log(m/n)=log m – log n log m=Klog m log m=log m log b	1	2	3	4	5

Sl.No.	Topics/Chapter	Sub-topics/Sections/Sub-sections						
		3.12	Principle of Mathematical Induction Theorem of Principle of finite Mathematical Induction	1	2	3	4	5
4	Mathematical Induction	4.1	Application of Mathematical Induction etc.	1	2	3	4	5
		4.3	x-y divides x-y for all positive integral values of n and other divisibility problems.	1	2	3	4	5
5	Permutations & Combinations	5.1	Definition of linear and circular permutations	1	2	3	4	5
		5.11	To find the number of permutations of a dissimilar things taken'r' at a time	1	2	3	4	5
		5.12	To prove nP=(n-1) P+r (n-1) P from the first principles	1	2	3	4	5
		5.13	To find number of circular Permutations of n different things taken all at a time.	1	2	3	4	5
		5.14	To find number of Permutations of n dissimilar things taken 'r' at a time when repetition of things is allowed any number of times.	1	2	3	4	5
		5.15	To find the number of Permutations of 'n' things taken all at a time when some of them are all alime and the rest are all different.	1	2	3	4	5
		5.2	Combinations-Definition	1	2	3	4	5
		5.21	To find the number of combinations of n dissimilar things taken 'r' at a time.	1	2	3	4	5
		5.22	To prove nC=nCn-r if nC=nCn=r+s	1	2	3	4	5
		5.22	To prove nC+nC 1=(n+1) C	1	2	3	4	5
6	Binomial Theorem	6.1	Binomial theorem for positive integral index Binomial coefficients and simple results on them, Numerically greatest ter	1	2	3	4	5

Sl.No.	Topics/Chapter	Sub-topics/Sections/Sub-sections						
		6.2	Binomial Theorem for rational index (statement only). Important particular cases of Binomial Expansion.	1	2	3	4	5
		6.3	Approximations by the use of Binomial Theorem	1	2	3	4	5
7	Partial fractions	7.1	Resolving f(x)/g(x) into partial fractions when g(x) contains non repeated linear factors.	1	2	3	4	5
		7.2	g(x) contains repeated and non repeated linear fractions only.	1	2	3	4	5
		7.3	g(x) contains repeated and non repeated linear fractions only.	1	2	3	4	5
		7.4	g(x) contains repeated non-repeated irreducible factors only. (Note: Number factors of g(x) should not exceed 4)	1	2	3	4	5
8	Exponential and logarithmic Series	8.1	1e I Expansion for real x.	1	2	3	4	5
		8.2	Log(1+x) expansion, condition on x (Note: Statements of the results and very simple problems such as finding the general term should only be given)	1	2	3	4	5

TRIGNOMETRY

Sl. No.	Topics/Chapter	Sub-topics/ Sections/Sub-sections						
1.	Graph & Periodicity	1.1	Variation of sin, cos, tan, Variation in values as changes periodicity and expreme values	1	2	3	4	5
		1.2	Graphs y=sin, y=tan	1	2	3	4	5
2.	Trigonometric ratios of Compound angles	2	Summation Formulae Sin(A+B), Cos(A+B), Tan(A+B)	1	2	3	4	5
3.	Trigonometric ratio of Multiple and submultiple Angles	3	Trigonometric ratios of 2A,3A,and A/2	1	2	3	4	5
4.	Transformations and Identities	4.1	Transformation from sum to products	1	2	3	4	5
		4.2	Transformation from products to sums	1	2	3	4	5
5.	Trigonometric Equations	5.1	General solution of Sin=4, Cos=k, Tan=k	1	2	3	4	5
		5.2	Solving simple Trigonometric Equations	1	2	3	4	5
6.	Inverse circular functions	6.1	Inverse of all the six trigonometric functions domains, ranges graphs.	1	2	3	4	5
		6.2	Solving simple Equations, Involving inverse Trigonometric functions	1	2	3	4	5
7.	Hyperbolif functions	7.1	Meaning of e expansion of the series [e]	1	2	3	4	5
		7.2	Definitions Domain and Range of Hyperbolic and Inverse Hyperbolic functions	1	2	3	4	5
		7.3	Addition formulae of Hyperbolic functions Singh (x+y), Cos. (x+y) etc	1	2	3	4	5
8.	Properties of triangles	8 1	Relation between the sides and angles of a triangle.	1	2	3	4	5
		8.2	Sine and Cosine rules-Nippers formulae Projection formulae	1	2	3	4	5
		8.3	Half angle formulae and area of a triangle	1	2	3	4	5
		8.4	Incircle and ex-circle of a triangle	1	2	3	4	5
9.	Heights & Distances	9.1	Angles of Elevation and Depression	1	2	3	4	5
		9.2	Problems involving one plane	1	2	3	4	5
10.	Complex numbers	10.1	Complex number as an ordered pair of real numbers. Representation in the form of a+ib. Real and imaginary part-equality of complex numbers.	1	2	3	4	5

Sl. No.	Topics/Chapter	Sub-topics/ Sections/Sub-sections						
		10.2	Fundamental operations on Complex Numbers	1	2	3	4	5
		10.3	Conjugate Complex Numbers	1	2	3	4	5
		10.4	Modulus and amplitude of a Complex Number	1	2	3	4	5
		10.5	Geometrical Representation of a Complex Number Argand Plane and Argand diagram	1	2	3	4	5
11.	Demovieris Theorem	11.1	Demovier's theorem for integral index and for rational index and for rational index.	1	2	3	4	5
		11.2	Nth root of unity and its geometrical representation	1	2	3	4	5
		11.3	Cube roots of unity.	1	2	3	4	5
		11 4	Expansion of Trigonometric functions, Sin n, Cos n as series expansion of Tan n and Cot n	1	2	3	4	5
		11.5	Expressing Sin and Cos interms of Sines and Cosines of multiples of	1	2	3	4	5

CO-ORDINATE GEOMETRY-2D

Sl. No.	Topics/Chapter	Sub-topics/Sections/Sub-sections						
1.	Locus	1.1	Definition of locus	1	2	3	4	5
		1.2	Equation to locus	1	2	3	4	5
		1.3	Illustrations	1	2	3	4	5
2.	Translation and Rotation of axes.	2.1	Translation of axed	1	2	3	4	5
		2.2	Rotation of axes	1	2	3	4	5
		2.3	Illustrations	1	2	3	4	5
3.	Straight line	3.1	Recapitudation of a) General equation of a straight line	1	2	3	4	5
			b) Forms of equation of a straight line i) slope intercept from ii) Intercepts form iii) Point-slope form iv) Two point form	1	2	3	4	5
		3.2	Normal form xCosx+ySinx=p	1	2	3	4	5
		3.3	Symmetric form	1	2	3	4	5
		3.4	To reduce the general education into different forms	1	2	3	4	5
		3.5	Point of intersection of two straight lines	1	2	3	4	5
		3.6	Family of straight lines passing through the point of intersection of two given lines	1	2	3	4	5
		3.7	Condition for concurrency of three straight lines	1	2	3	4	5
		3.8	Angle between two intersecting lines. Condition for perpendicularity and parellelisom.	1	2	3	4	5
		3.9	Length of perpendicular form a point to a straight line, distance between two parallel lines	1	2	3	4	5
		3.10	The medians of a triangle are concurrent	1	2	3	4	5
		3.11	The altitudes of a triangle are concurrent	1	2	3	4	5
		3.12	The internal bisectors of the angles of a triangle are concurrent	1	2	3	4	5
		3.13	The perpendicular bisectors of the sides of a triangles are concurrent	1	2	3	4	5

Sl. No.	Topics/Chapter	Sub-topics/Sections/Sub-sections						
4.	Pair of straight lines	4.1	Ax+2hxy+by=0 a pair of lines through the origin	1	2	3	4	5
		4.2	Angle between the lines ax+2hxy+by=0 conditions for coincidence perpendicularities	1	2	3	4	5
		4.3	Bi-sector of the angles between the lines L=0, L-0	1	2	3	4	5
		4.4	Combined equation of the bisectors of the angles between the lines ax+2hy+by-0	1	2	3	4	5
		4.5	If the general equation of the second degree in x & y i.e.ax+2hxy+by+2gx+2fy+0=0 represents a pair of straight lines then abc+2fgh-af-bg-ch=0, h>ab, g>ac and f>bc.	1	2	3	4	5
		4.6	Converse of 4.5 without proof	1	2	3	4	5
		4.7	If S=ax+2hxy+by+2gx+2fy+c=0 represents a pair of straight lines then ax+2hxy+by=0 represents the lines through the origin, parallel to the above lines. Angle between the lines S=0 condition for these line to be (1) Parallel (ii) Perpendicular	1	2	3	4	5
		4.8	Point of Intersection of the lines S=0	1	2	3	4	5
		4.9	Homogenization of the second degree equation with a first degree equation in x any y.	1	2	3	4	5

CO-ORINATE GEOMETRY-3D

Sl. No.	Topics/Chapter	Sub-topics/Sections/Sub-sections						
1.	Co-ordinates	1.1	Co-ordinates of a point Distance between two points in space	1	2	3	4	5
		1.2	To find the C0-ordinates of a point which divides the joint of two points (x Internally in the ration m:n-Centroid of a triangle and tetrahedron	1	2	3	4	5
2	Direction consines and rations	2.1	Direction consines of a line, relation between the direction consines 1+m+n=1	1	2	3	4	5
		2.2	Direction ratios of a line-to find the direction consines when direction ratios are given-Angle between lines, condition for perpendicularity and parallelism.	1	2	3	4	5
		2.3	Projection of line-length of the projection Projection of join of the two points on a line whose Direction cosiness are 1,m,n.	1	2	3	4	5
3.	Plane	3.1	General equation of first degree in x,y,z represents a plane-reducing general equation into intercepts form. Equations of any plane through (x,y,z) is A(x-x)+B(y-y)+C9z-z)=0	1	2	3	4	5
		3.2	Equation of the plane-normal form	1	2	3	4	5
		3.3	Direction cosiness to the normal to the plane Ax+By+Cz+D=0	1	2	3	4	5
		3.4	The equation of the plane passing through three points.	1	2	3	4	5
			CALCULUS					
4.	Function, Limits and Continuity	1.1	Function, Domain and Range of Function-Algebric, Trigonometric, Inverse Trigonometric, Hyperbolic, Step-function and construction of Graphs for logx, [e],[x],[x].	1	2	3	4	5
		1 2	Concepts of intervals and neighbourhood	1	2	3	4	5
		1.3	Limits-Definition of right hand limit, left hand limit, limit. Limits of f+g, f/g, fog (without proofs)	1	2	3	4	5
		1.4	Standard limits 1) Lt	1	2	3	4	5

Sl. No.	Topics/Chapter	Sub-topics/Sections/Sub-sections						
2.	Differentiation	1.5	Continuity-Definition and simple illustration	1	2	3	4	5
		2.1	Introduction-Definition	1	2	3	4	5
		2.2	Differentiation of a function at a point and on an Interval-Derivative of a function-Differention of Sum, differences, product and quotient of functions. Differentiation of algebraic, circular, exponential, Logarithmic function.	1	2	3	4	5
		2.3	Derivatives of composite, implicit, parametric, inverse Circular, hyperbolic and inverse hyperbolic functions	1	2	3	4	5
		2.4	Logarithmic differentiation, Derivation of a function with respect to another function.	1	2	3	4	5
3.	Successive differentiations	3.1	Successive differentiation-Introduction of nth derivative of 1(ax+b, log(ax+b), e Sin(ax+b), Cos(ax+b)	1	2	3	4	5
4.	Application of Derivative	3.2	Leibnitz theorem and its application	1	2	3	4	5
		4.1	Infinitesimals-Differentials	1	2	3	4	5
		4.2	Errors and approximations	1	2	3	4	5
		4.3	Geometrical interpretation of a derivative	1	2	3	4	5
		4.4	Equations of tangent, normal at a point on the line.	1	2	3	4	5
		4.5	Lengths of tangent, normal sub-tangent, sub-normal At a point	1	2	3	4	5
		4.6	Angle between two curves, orthogonality	1	2	3	4	5
		4.7	Derivate as a rate measurer	1	2	3	4	5
		4.8	Increasing and decreasing functions	1	2	3	4	5
		4.9	Maxima and minima	1	2	3	4	5

MATHEMATICS-II(A)
(ALGEBRA, VECTOR ALGEBRA AND PROBABILITY)
DISTRIBUTION OF SYLLABUS INTO TOPICS/SUB-TOPICS

Sl.No.	Topic/Sub-topic	No. of periods				
	1. QUADRATIC EXPRESSIONS:					
1.1	Quadratic expressions, equation in one variable-extreme values-changes in sign and magnitude-Quadratic in equation.	1	2	3	4	5
	2. THEORY OF EQUATIONS	1	2	3	4	5
2.1	Relation between the roots and coefficients in any equation					
2.2	Soving the equations when two or more roots of it are connected By certain relations.	1	2	3	4	5
2.3	Equation with real coefficients imaginary roots occur in Conjugate pairs and its consequences	1	2	3	4	5
2.4	DesCarte's rule of signs	1	2	3	4	5
2.5	To find the sum of any assigned powers of roots of the equation	1	2	3	4	5
2.6	Transformation of equations-reciprocals equations	1	2	3	4	5
2.7	Cubic equations-Cardan's solution	1	2	3	4	5
2.8	Biquadratic equation-Ferrari and DesCarte's solution	1	2	3	4	5
	3. MATRICES					
3.1	Definition-Types of Matrices-Equality-Addition. Commulative and associate properties of addition.	1	2	3	4	5
3.2	Scalar multiplication of a matrix-additive inverse and identity-Multiplication of matrice-Non-commutativity Associative and Distributive laws on multiplication.	1	2	3	4	5
3.3	Transpose of a matrix-properties	1	2	3	4	5
	Symmetric and skew symmetric matrices	1	2	3	4	5
3.4	Inverse of a matrix-determinant of Matrix-Singular and Non-singular Matrices, minor, co-factor of one element.	1	2	3	4	5
3.5	Properties of determinants-Adjoint of a Matrix	1	2	3	4	5
3.6	Solution, of simultaneous linear equations in two and three variables By cramer's rule, matrix inversion method and Gauss-Jordan method, Consistency and inconsistency of simultaneous equations.	1	2	3	4	5
	ADDITION OF VECTORS:					
4.1	Introduction of vector as an ordered triad of real numbers-Representation of vector as a directed line segment-Free and localized Vectors	1	2	3	4	5
4.2	Classification of vectors-Equal, negative Collinear or parallel, like, Unlike vectors, co-initial vectors, coplanar and Non-coplanar vectors, position vector Unit vectors etc.	1	2	3	4	5
4.3	Addition of Vectors-Parallelegram and triangle laws-properties of additions-Subtraction of vectors	1	2	3	4	5
4.4	Multiplication of a Vector by a scalar	1	2	3	4	5
4.5	Angle Between two vectors-vector of the point of division-Concurrency of Medians of a triangle by vector method	1	2	3	4	5

Sl.No.	Topic/Sub-topic	No. of periods				
4.6	Liner combinations of vectors-Linearly dependent and linearly Independent vectors	1	2	3	4	5
4.7	Components of Vectors in three dimensions-Direction cosines, Modulus of a vector-Right and Left hand system,	1	2	3	4	5
4.8	Vector equation of line and plane in parametric form Collinearly of Three points and coplanarity of four points	1	2	3	4	5
	5. MULTIPLICATION OF VECTORS.					
4.8	Definition of scalar or dot product of two vectors its geometrical Interpretation-and Orthogonal projection off	1	2	3	4	5
4.9	Properties of Scalar product-Commutative, distributive law	1	2	3	4	5
4.10	Proof by vector method of	1	2	3	4	5
	(i) angle in a semi circles is a right angle	1	2	3	4	5
	(ii) laws of cosines and projection formulas in a triangle (ii) laws of cosines and projection formulas in a triangle	1	2	3	4	5
4.11	Vector equation of a plane in the normal form two types-Angle between two planes	1	2	3	4	5
4.12	Vector product of two vectors-Non-commutativity Voctor product is distributive over addition	1	2	3	4	5
5 3	Analytic expressions for scalar products in term angle between two vectors some identities such as (a+b) etc	1	2	3	4	5
5.4	Proof of some geometrical and trigonometrical theorems by Vector Methods	1	2	3	4	5
5.5	Vector equations of a plane in the normal form – angle between two planes.	1	2	3	4	5
5.6	Vector product of two vectors.	1	2	3	4	5
5.7	Vector product among Sine of the angle between two vectors-Unit vector perpendicular to the pair of vectors.	1	2	3	4	5
5.8	Vector area of parallelgram and a triangle-vector method proofs (i) sine rule (ii) sin(A+B)-sin A cos B+ cos A sin B and (iii) area of triangle	1	2	3	4	5
5.9	Scalar triple product-Geometrical inter-pretation-Coplanarity of three vectors, And deductions –volume of a tetra hedran	1	2	3	4	5
5.10	Vector area of plane in different forms-skew lines simple problems	1	2	3	4	5
5.11	Vector triple product and its result	1	2	3	4	5
5.12	Products of Four vectors-scalar and vector product of four products	1	2	3	4	5
	6.PROBABILITY					
6.1	Random experiment, random event, elementary events, exhaustive Events, mutually exclusive events etc.,	1	2	3	4	5
6.2	Classical definition-relative frequency approach-sample space, sample Events, addition theorem	1	2	3	4	5
6.3	Dependent and independent events-Multiplication theorems	1	2	3	4	5

Sl.No.	Topic/Sub-topic	No. of periods				
	7. RANDOM VARIABLE & DISTRIBUTIONS					
7.1	Random variable, Distributive functions, probability distributive functions-Mean and Variance of random variable	1	2	3	4	5
7.2	Theoretical, discrete distribution like Binomial,. Poisson, distribution-Mean and variance of above distributions (without derivation)	1	2	3	4	5

MATHEMATICS – II(B)
(CO-ORDINATE GEOMETRY AND CALCULUS)
DISTRIBUTION OF SYLLABUS INTO TOPICS/SUB-TOPICS

S.No.	Topic/Sub-Topic	No. of Periods				
	1.CIRCLES:					
1.1	Equation of a circle-Standard form-Standard form-centre and radius- Equation of circle with a given line segment as diameter- Equation of circle through three non-collinear points- Parametric equations of a circle.	1	2	3	4	5
1.2	Position of a point in the plane of the circle- Power of a point-Def.of a tangent-Length of tangent.	1	2	3	4	5
1.3	Position of a straight line in the plane of the circle-condition for a straight line to be a tangent-chord joining two points on al circle-equation of the tangent at a point on the circle-point of a contact - Equation of normal.	1	2	3	4	5
1.4	Chord of contact Pole, Polar-conjugate points and conjugate line-Equation of chord given mid point.	1	2	3	4	5
1.5	Relative positions of two circles-circles touching each other, externally, internally, number of common tangents-points of similitude-Equation pair of tangents from an external point.	1	2	3	4	5

2. SYSTEMS OF CIRCLES:

S.No.	Topic/Sub-Topic	No. of Periods				
2.1	Angle between two intersecting circles-conditions for orthogonality	1	2	3	4	5
2.2	Radical axis of two circles-properties-Common chord and common tangent of two circles, Radical cente.	1	2	3	4	5
2.3	Coaxal system of circles-Equation of the coaxal system in the simplest form-Limiting points of a coaxal system.	1	2	3	4	5
2.4	Orthogonel system of a coaxal system of circles.	1	2	3	4	5

3. PARABOLA

S.No.	Topic/Sub-Topic	No. of Periods				
3.1	Conic section-parabola-Equation of parabolal in standard form – Different forms of parabola-parametric equation.	1	2	3	4	5
3.2	Equation of tangent and normal at a point on the parabola (certesian and parametric) Condition for a straight line to be a tangent.	1	2	3	4	5
3.3	Pole and Polar Finding the polel of la given line and Vice Versa.	1	2	3	4	5

4. ELLIPSE

S.No.	Topic/Sub-Topic	No. of Periods				
4.1	Equation of Ellipse in standard form , parametric equations.	1	2	3	4	5
4 2	Equation of tangent and normal at a point on the ellipse (Certesian and Parametric) condition for a straight line to be a tangent.	1	2	3	4	5
4.3	Pole and Polar- Finding the pole of a given linc and Vice Versa.	1	2	3	4	5

5. HYPERBOLA

S.No.	Topic/Sub-Topic	No. of Periods				
5.1	Equation of hyperbola in standard form – parametric equations Rectangular Hyperbola.	1	2	3	4	5
5.2	Equation of tangent and normal at a point on the hyperbola (Caertesian and Parametric) condition for a straight line to be a tangent. Asymptotes.	1	2	3	4	5
5.3	Pole and Polar-Finding the pole of a given line and Vice Versa.	1	2	3	4	5

6. POLAR COORDINATES

S.No.	Topic/Sub-Topic	No. of Periods				
6.1	Polar coordinates-Relation between polar and Cartesian coordinates-Distance between two points – Area of a triangle.	1	2	3	4	5
6.2	Polar equation of a straight line, circle and a conic.	1	2	3	4	5

CALCULUS

7. PARTIAL DIFFERENTIATION

S.No.	Topic/Sub-Topic	No. of Periods				
7.1	Partial derivatives-First and second order only.	1	2	3	4	5
7.2	Homogeneous functions – Euler's theorem on homogenous function-simple application.	1	2	3	4	5

8. METHODS OF INTERGRATION

S.No.	Topic/Sub-Topic	No. of Periods				
8.1	Integration as the inverse process of differentiation-standard forms-properties of integrals	1	2	3	4	5
8.2	Integration by method of substitution covering algebraic, trigonometric and exponential Functions. Integration by parts-logarithmic functions-inverse trigonometric	1	2	3	4	5
8.3	Integration of rational functions using partial fraction;	1	2	3	4	5
8.4	Integrals of the following types of functions.	1	2	3	4	5

9. DEFINITE INTEGRAL

S.No.	Topic/Sub-Topic	No. of Periods				
9.1	Interpretation of definite integral as an area-Fundamental theorem of calculus(without proof) properties of definite integrals-Evaluation of definite integral	1	2	3	4	5
9.2	Reduction formulae for evaluation of integrals	1	2	3	4	5

10.NUMERICAL INTEGRATION

S.No.	Topic/Sub-Topic	No. of Periods				
10.1	Determination of plane areas involving second degree curves-Areas under the curve sinx and cosx	1	2	3	4	5
10.2	Trapezoidal Rule and simpson's Rule(without proof)-simple applications.	1	2	3	4	5

11. DIFFERENTIAL EQUATIONS

S.No.	Topic/Sub-Topic	No. of Periods				
11 1	Formation-general and particular solution and primitives-Degree and order of an Ordinary differential equation.	1	2	3	4	5
11 2	Solutions of the first order and first degree equations of the following types					
	(i) Variables separable	1	2	3	4	5
	(ii) Equations of the type	1	2	3	4	5
	(iii) Equations of the type (Linear) Dy/dx+P,Y=Q where P Q are functions of X	1	2	3	4	5

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